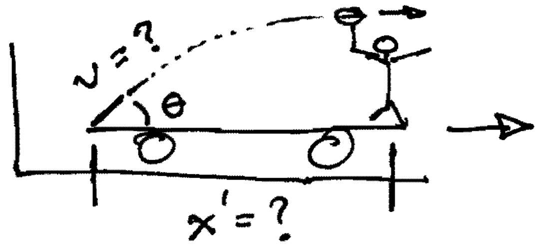
5.



Do the calculation in the frame of the train car. Then it is just a standard projective motion problem, Forget that it takes place on a train. at the maximum height, $\vec{v} = v_{0x}\hat{x}$ only (purely horizontal).

use $v^2 = v_0^2 + 2a\cancel{x}y$.

$a = -g$
$y = h$
$v_0 \to v_{0y} = v_0 \sin\theta$.
$v = 0$ at top

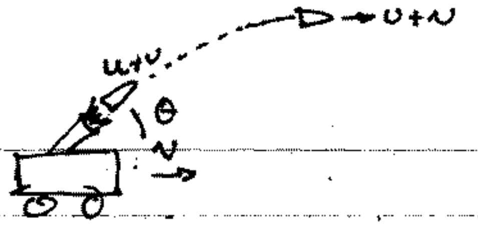
$v^2 = 0 = v_0^2 \sin^2\theta - 2gh \to v_0 = \frac{\sqrt{2gh}}{\sin\theta}$

(b). how far? $x' = v_{0x} \cdot t$ and t you get from ~~v_{y}~~ $v_y = v_{0y} - gt$ where $v_y = 0$

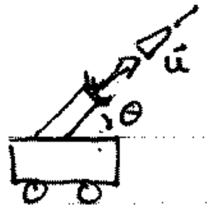
$\to v_{0y} = v_0 \sin\theta = gt$
or $t = \frac{v_0 \sin\theta}{g} = \sqrt{\frac{2h}{g}}$

So $x = v_0 \cos\theta \cdot t = \frac{\sqrt{2gh}}{\sin\theta} \cdot \cos\theta \cdot \sqrt{\frac{2h}{g}} = \frac{2h}{\tan\theta}$.

1.4



"Floor frame"

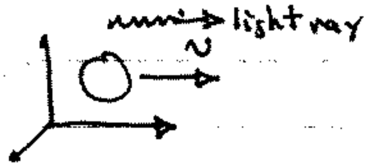


"Rest frame"

The best frame is the one with the cart at rest. Then if $\underline{\theta = 90^\circ}$ the shell just goes straight up and comes back down. In the "floor frame" both cart and cannon ball have velocity v, then when fired the cannon ball has velocity $u+v$, only if u has x component 0 will it arrive back at the cannon.

1.9. Guess that velocity through the ether is $\sim 3\times10^4$ m/s.

(a). What is the observed speed of a light wave parallel to $\vec{v}$? Since $v_{light} = c$ in the ether frame if you are going in the same direction $v_{obs} = c - v$ (seems slower)

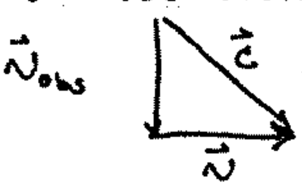


$$v_{obs} = 2.9979\times10^8 \text{ m/s} - .0003\times10^8 \text{ m/s}$$
$$= 2.9976\times10^8 \text{ m/s}.$$

(b). anti-parallel to c? then $v_{obs} = c + v$

or $v_{obs} = (2.9979 + .0003)\times10^8 \text{ m/s} = 2.9982\times10^8 \text{ m/s}$.

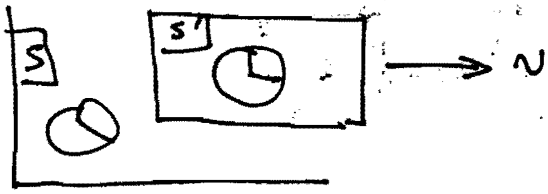
(c) Perpendicular to v: use vector relation $\vec{v}_{obs} = \vec{c} - \vec{v}$



so $c^2 = v_{obs}^2 + v^2$ and $v_{obs} = \sqrt{c^2 - v^2}$

$\cong 2.9979\times10^8$ m/s.

21. (a). what must $v=$ if we lose 1 sec/day?



Clock S' runs 1 second/day slower than S. There are 24×3600 seconds in 1 day So: the rate of slowing is 1 part in 86400, which is 1.16×10^{-5}.
So $\Delta t' = \gamma\Delta t$ such that $\Delta t' = 1. + 1.16\times10^{-5}$
or $\gamma = 1 + 1.16\times10^{-5}$

$\beta = \sqrt{1-\frac{1}{\gamma^2}} = 4.81\times10^{-3}$ or $v = 1.44\times10^{6}$ m/s.

1 minute per day → 1 part in 1440

then $\beta = 3.72\times10^{-2}$ or $v = 1.12\times10^{7}$ m/s

1.20. A vehicle travels at $v = 100\,000\ m/s$. (or $v = 10^5\ m/s$). How much time will clocks gain or lose relative to earth in 1 day?

$$v = 10^5\ m/s\ ,\quad c = 3\times10^8\ m/s \Rightarrow \beta = 3.33\times10^{-4}$$

~~[illegible]~~. With $\beta = 3.33\times10^{-4}$, then

$$\gamma = 1.000000055556$$

$$(\text{or } \gamma - 1 = 5.5556\times10^{-8}).$$

in 1 day; $\Delta t = \gamma\Delta t'$

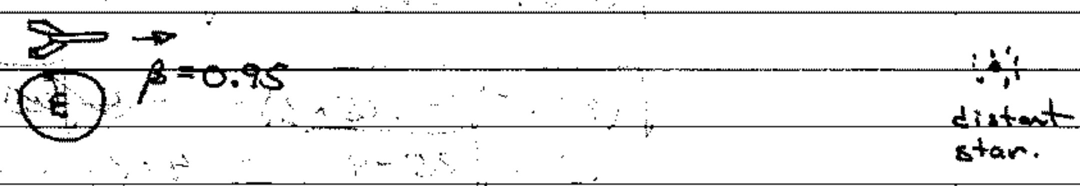
the change is $(1-\gamma)\Delta t = (1-\gamma)\,86400\,s \simeq 4.8\times10^{-3}\,s$

another way is to use the binomial expansion

$$(1+x)^n \approx 1 + nx$$

or $(1-\beta^2)^{-1/2} \approx 1 + \frac{1}{2}\beta^2$ so the clocks will be different by $\frac{1}{2}\beta^2 \approx 5.55\times10^{-8}$ which gives you the same answer.

1.22.



Earth time is 80 years past. How much has the explorer aged?

$\beta = 0.95 \Rightarrow \gamma = 3.2$

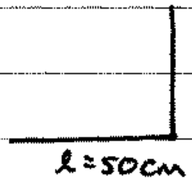
Treat the two halves of the trip separately

$$\Delta T_{e_1} = \gamma\,\Delta T_{R_1} \qquad \text{and} \qquad \Delta T_{e_2} = \gamma\,\Delta T_{R_2}$$

(we don't really need to do this to solve it, but formally it is more correct).

$$2\Delta T_e = 80 \text{ years} = \gamma\,(2\Delta T_R) \Rightarrow 2\Delta T_R = \frac{80}{\gamma} = \frac{80}{3.2} \simeq 25 \text{ years}$$

1.17



What is the expected phase shift ΔN, assuming: $\lambda = 590$ nm and $v = 3\times10^4$ m/s.

Use 1.11: $\Delta N = \frac{2\ell\beta^2}{\lambda}$ $\quad \beta = 1\times10^{-4}$ $\quad \ell = .5$ m
$\lambda = 590\times10^{-9}$ m

$\Rightarrow \Delta N = \frac{2\times.5\times10^{-8}}{590\times10^{-9}} = .017$ → very small.

1.26 π-mesons in the lab. $\quad v = 0.8c \quad \beta = .8$

(a). $\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-.8^2}} = \frac{1}{\sqrt{1-.64}} = \frac{1}{\sqrt{.36}} = \frac{1}{.6} = 1.667$

(b). $(t_{1/2})_\pi = 1.8\times10^{-8}$s $\quad$ what is $(t_{1/2})_{lab}$?
$(t_{1/2})_{lab} = \gamma (t_{1/2})_\pi = 3\times10^{-8}$s

(c). $N_0 = 32000$. $\quad \ell = 36$ m. $\quad \Delta t = \frac{\ell}{v} = \frac{36\text{m}}{.8\times3\times10^8\text{ m/s}} = 1.5\times10^{-7}$s

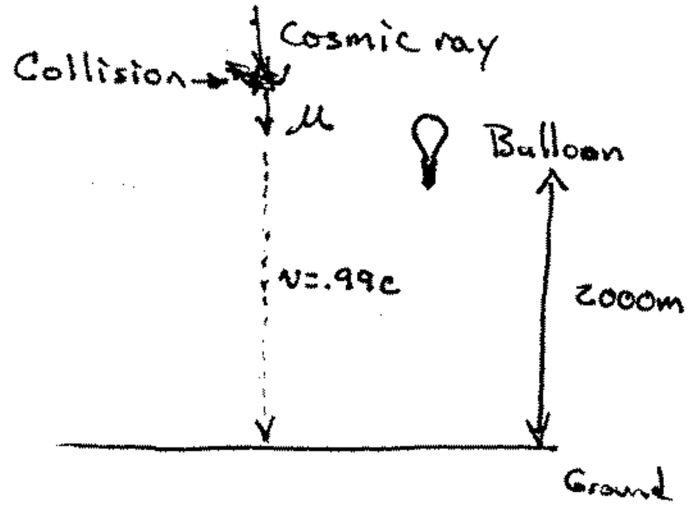
how many $\frac{1}{2}$-lives is 1.5×10^{-7}s?
$\#\,\frac{1}{2}$ lives $= \frac{\Delta T}{(t_{1/2})_{lab}} = \frac{1.5\times10^{-7}}{3\times10^{-8}} = 5$

be sure to use lab value.

So $N_{left} = N_0/2^5 = \frac{N_0}{32} = 1000$.

(d) Ignoring time dilation $\#\,\frac{1}{2}$lives $= \frac{1.5\times10^{-7}\text{s}}{1.8\times10^{-8}\text{s}} = 8.333\ldots$
an N_{left} would be $\frac{32000}{2^{8.33}} = 99$.
A factor of 10 fewer!

1.27 Muons.



$(t_{1/2})_\mu = 1.5 \times 10^{-6}$ s

In the balloon: $\#\mu$/hour = 650
what is $\#\mu$/hour on the ground?

If $\beta = 0.99$ $\quad \gamma = \frac{1}{\sqrt{1-.99^2}} = 7.09$

So $(t_{1/2})_{ground} = 7.09 \times 1.5 \times 10^{-6}$ s $= 1.06 \times 10^{-5}$ s. $\quad (\gamma \times (t_{1/2})_\mu)$

How long to travel 2000m? $\quad \Delta T = \frac{2 \times 10^3 \text{m}}{.99 \times 3 \times 10^8 \text{ m/s}} \simeq 6.73 \times 10^{-6}$ s

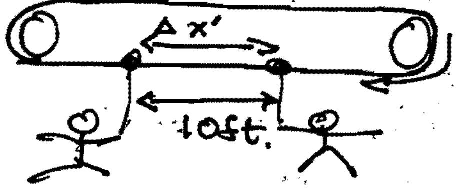
How many ½ lives is ΔT?

$\#$ ½ lives $= \frac{6.73 \times 10^{-6} \text{s}}{1.06 \times 10^{-5} \text{s}} = 0.635$

$\gamma (t_{1/2})_\mu$

So rate on the ground $= \frac{1}{2^{.635}} \times 650$/hr $= .644 \times 650 \simeq 419$ / hour

30.



$\rightarrow v = .5c$ $\quad \gamma = \frac{1}{\sqrt{1-.5^2}}$

$= \frac{2\sqrt{3}}{3} = 1.155$

This is tricky, here $\Delta x = 10\,ft$
but that is the contracted value of $\Delta x'$
i.e. $\Delta \cancel{x} = \frac{\Delta x'}{\gamma}$ so $\Delta x' = \gamma \Delta x$

or $\Delta x' = 11.55\,ft.$
this can also be seen by working out the Lorentz transformations for the events E_1, E_2 where $E_1: (0,0)$. $E_2: (10ft, 0)$ in S
in S': $x_1' = \gamma(0-0) = 0$
$x_2' = \gamma(10ft - v\cdot 0) = \gamma \cdot 10ft = 11.55ft.$

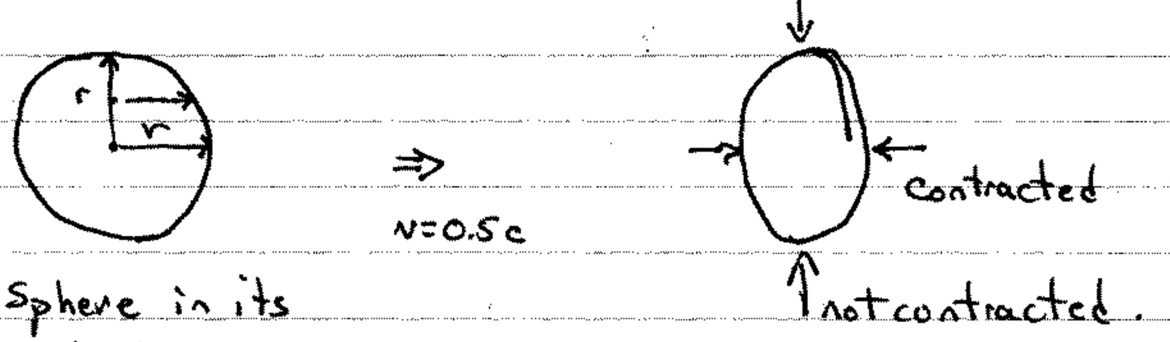
Also note that according to a viewer on the belt, the marks are not made simultaneously:
$t_1 = t_2 = 0$
$t_1' = \gamma(t_1 - \frac{v x_1}{c^2}) = 0$
$t_2' = \gamma(t_2 - \frac{v x_2}{c^2}) = \gamma(0 - \frac{(.5c)\times 10ft}{c^2})$

in horrible units, c is about 1ft/ns.
so $t_2' = 1.155\times(0 - \frac{.5\cdot 10}{1\cdot 1\,\frac{ft^2}{ns^2}}\,\cancel{\frac{ft}{ns}}\cdot 10ft)$

$= -1.155\cdot 5ns = -5.78\,ns.$
$\Delta t' \neq 0$!

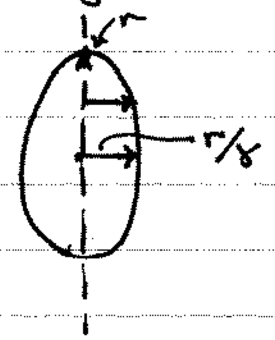
This is exactly the same as the snake problem.

1.31



Sphere in its
rest frame.

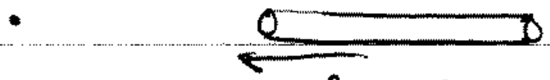
In the ground or lab frame, the sphere is contracted along its direction of motion. How much?



think of each point on a circle. The equation that describes a circle is $x^2+y^2=r^2$. y is unaffected but each x gets contracted to $\frac{x}{\gamma}$. $\gamma = 1.15$

So the shape looks like a flattened ball, whose cross section has the shape described by $\gamma^2x^2+y^2=r^2$

1.32. Pions in their rest frame. $t_{1/2} = 1.8\times10^{-8}s$.



$l_0 = 36m$

How far does the lab move? $l_\pi = \frac{l_0}{\gamma} = \frac{l_0}{1.667} = 21.6m$

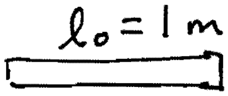
How long does this take?

$$\Delta T_\pi = \frac{l_\pi}{v} = \frac{\cancel{\gamma}\times l_0/\gamma}{.8c} = \frac{1.667^{-1}\times 36m}{.8c} = \frac{21.6m}{.8\times3\times10^8 m/s} = 9\times10^{-8}s$$

How many ½ lives? $\#\tfrac{1}{2}\text{ lives} = \frac{\Delta T_\pi}{(t_{1/2})_\pi} = \frac{9\times10^{-8}s}{1.8\times10^{-8}s} = 5$

Just like before, and

$\#\text{Left} = \frac{N_0}{2^5} = \frac{32000}{2^5} = 1000$. As it had better!

33. (a) 

$\rightarrow v = .8c$

Then in S, the apparent length $l = \frac{l_0}{\gamma}$

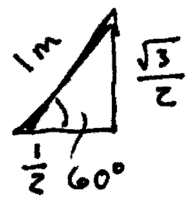
$\gamma = \frac{1}{\sqrt{1-.8^2}} = \frac{5}{3}$ so $l = .6 \cdot 1m = .6m$

(b) 

$\rightarrow .8c$ since the stick is $\perp$ to the motion, it is not contracted.

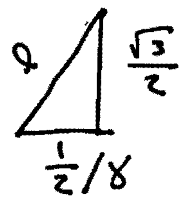
So $l = l_0 = 1.0m$.

(c). in S':



only the base of the triangle is contracted. height is unchanged.

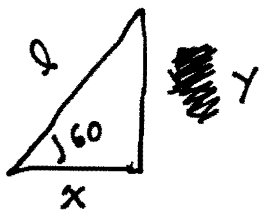
in S:



base $x = \frac{1}{2}m/\gamma = .6 \times .5m$
$= .3m$

So $l = \sqrt{\frac{3}{4} + (.3)^2} = .917\ m$.

(d) in S:



here use $\tan(60°) = y/x$

or $x = \frac{y}{\tan 60} = \frac{y}{\sqrt{3}}$

$x = x'/\gamma$ or $x' = \gamma x = \frac{5}{3}x$

in S', we know $l' = l_0 = 1m$, $y = y'$

So we can solve for x' and x:

$l_0 = \sqrt{x'^2 + y^2}$ or $l_0^2 = (\frac{5}{3})^2 x^2 + 3x^2 = 1m$

so $x = \frac{1}{\sqrt{(\frac{5}{3})^2 + 3}} = .416m$

so apparent $l = \sqrt{(.416m)^2 + (.416 \times \sqrt{3})^2} = 2 \times .416 = \underline{.832m}$